UNIVERSITY OF MIAMI **Experimentally Calibrated COMSOL Multiphysics Model for Glucose-Stimulated Insulin Release:** MILLER SCHOOL Effect of Encapsulation and Device Geometry on First- and Second Phase Insulin Response of MEDICINE



FEM-Based GSIR Model: Concept and Implementation

Computational modeling is done with our recently developed and calibrated COMSOL Multiphysics model of insulin secretion in avascular islets) [1-3]. Insulin-secreting β -cells are assumed to act as sensors of both the local glucose concentration and its change:

- c_{gluc} (local glucose concentration) \rightarrow second-phase insulin response
- $\partial c_{gluc} / \partial t$ (change in local c_{gluc}) \rightarrow first-phase insulin response

All nutrient consumption and hormone release rates are assumed to follow Hill-type sigmoid dependences on local concentrations. Parameterization has now been done with experimental GSIR perifusion data for both human [1, 2] and murine islets (including alginate-encapsulated ones) [3].

Present Model of Insulin Secretion (Avascular Islets)



Schematic concept of the present model of glucose-stimulated insulin secretion in β-cells. It was conceived within the general framework of sigmoid proportional-integral-derivative (SPID) controllers. Response is determined by the local glucose concentration, c_{gluc} , and its rate of change, $\partial c_{gluc}/\partial t$, but it is also influenced by the local oxygen concentration, c_{oxy} , as the availability of oxygen can be a major limiting factor in avascular islets. A total of four concentrations are modeled for glucose (c_{gluc}) , oxygen (c_{oxy}) , and 'local' and released insulin (c_{insL}, c_{ins}) , respectively. All hormone secretion and nutrient consumption kinetics are coupled with diffusive and convective transport via a fluid dynamics model. The model can account for both first- and second-phase insulin secretion, and can be used for arbitrary geometries and glucose stimulation sequences.

Model	Var.	C _{Hf}	n	R _{max}	Comments
R _{oxy} , oxygen	C _{oxy}	1 µM	1	-0.034	Cut to 0 below critical value,
consumption, base	$R_{oxy} = R_{\max, oxy}$	$\frac{c_{oxy}}{c_{oxy} + C_{Hf,oxy}} \cdot \varphi_{o,g}(c_{gluc}) \cdot \delta(c_{gluc})$	$C_{oxy} > C_{cr,oxy}$	mol/m³/s	c _{oxy} < C _{cr,oxy} .
R _{oxy} , oxygen	C _{gluc}	7 mM	2.5	N/A	Due to increasing metabolic demand;
consumption, $\varphi_{\rm o,g}$		(N		parallels second-phase insulin
metabolic part	$\varphi_{o,g}\left(c_{gluc}\right) = \varphi$	$\phi_{sc}\left(arphi_{base} + arphi_{metab} rac{c_{gluc}^{n_{im2,glu}}}{c_{gluc}^{n_{im2,glu}} + C_{H}^{n}} ight)$	ins2,gluc ff,ins2,gluc		secretion rate.
R _{gluc} , glucose	C _{gluc}	10 µM	1	-0.028	Contrary to oxygen, has no significant
consumption	$R_{gluc} = R$	$P_{\max,gluc} \frac{C_{gluc}}{C_{gluc} + C_{Hf,gluc}}$		mol/m³/s	influence on model results.
R _{ins,ph2} , insulin	C _{gluc}	7 mM	2.5	3×10 ⁻⁵	Total secretion rate is modulated by
secretion rate, second-		$c^{n_{ins^2,gluc}}$		mol/m³/s	local oxygen availability (last row).
phase	$R_{ins,ph2} = R$	$\sum_{max,ins2} \frac{c_{gluc}^{n_{ins2,gluc}}}{c_{gluc}^{n_{ins2,gluc}} + C_{Hf,ins2,gluc}^{n_{ins2,gluc}}}$			
R _{ins,ph1} , insulin	$\partial c_{gluc} / \partial t$	0.03 mM/s	2	21×10 ⁻⁵	Modulated via equation below to
secretion rate, first-				mol/m³/s	have maximum sensibility around $c_{\rm gluc}$
phase	<i>R</i> = <i>R</i>	$\frac{\left(\frac{\partial c_{gluc}}{\partial t}\right)^{n_{ins1,gluc}}}{\sum_{i=1}^{n_{ins1,gluc}} + Ct_{Hf,ins1,gluc}^{n_{ins1,gluc}}} \cdot \sigma_{i1,g} \left(e^{\frac{c_{gluc}}{2}} + Ct_{Hf,ins1,gluc}^{n_{ins1,gluc}} \right)$	c.)		= 5 mM and be limited at very large
	$-\lim_{m \to \infty} \lim_{n \to \infty} \int \frac{\partial c_{gli}}{\partial t}$	$\frac{uc}{dc} \int_{0}^{n_{ins1,gluc}} + Ct_{Hf,ins1,gluc}^{n_{ins1,gluc}}$	- guuc)		or low c _{gluc} .
Insulin secretion rate,	C _{oxy}	3 μΜ	3	N/A	To abruptly limit insulin secretion if
$\varphi_{\mathrm{o,g}}$ oxygen depend.	$\sigma_{_{i1,g}}(c_{_{gluc}}$	$(z) = rac{4c_{gluc}^{4}C_{m}^{4}}{\left(c_{gluc}^{4}+C_{m}^{4} ight)^{2}}$			c _{oxy} becomes critically low.

Oxygen concentrations used: $c_{\text{atm}} = 0.200 \text{ mol/m}^3$ (0.2 mM; $pO_2 \approx 140 \text{ mmHg}$, 18% O_2 ; normal culture 95% air, 5% CO₂; 37°C); 'tissue' (slight hypoxia: c_{in} = 0.06 mol/m³ (0.06 mM; pO₂ ≈45 mmHg, 5% O₂). Human islets: $k_{insl} = 0.003 \text{ s}^{-1} \rightarrow \text{Mouse islet}$: $k_{insl} = 0.006 \text{ s}^{-1}$.

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Results – Fit of Human and Murine Islet Data Human Islets – Dynamic Perifusion, Staircase Experiment 0.08 0.02

GSIR (perifusion) data (human islets) used for model parametrization [1]. Experimental data (blue disks, •) are for human islets (Henquin et al. Diabetes 2006, 55, 3470) and values calculated with the present model (red line, —) after final calibration are shown superimposed on the same time-scale [1].



GSIR (perifusion) data for un-encapsulated (naked or free) and encapsulated islets perifused in parallel and their fit with the present computational model [3]. Experimental data (symbols) were determined in dynamic perifusion with frequent sampling (every minute) using free and alginateencapsulated ($d \approx 800 \,\mu$ m) islets (~50 IEQ, average of three experiments in duplicates) in response to a single glucose step as show. Corresponding model-calculated insulin outflow is shown for capsule thicknesses of 0 (free islets) and $I_{caps} = 150 \ \mu m$ assuming islet sizes as shown in figure with geometry and following a 2D to 3D conversion with a total islet volume scaled to 50 IEQ (see text).



Model-predicted effect of capsule thickness on insulin secretion profile [3]. Calculated insulin outflow is shown in response to a stepwise glucose stimulation with the current computational model for two encapsulated islets ($d = 100 \& 150 \mu m$) as a function of increasing capsule thickness (I_{caps}) from 0 (free islets) and 50 to 350 µm. Inset (right) shows an illustration of the main geometric setup used for the current model with fluid flow from left to right, and 100 (top) and 150 (bottom) μm islets.





Insulin secreting efficiency of differently sized islets. Data shown are *islet efficiency* [calculated as the total insulin release per islet volume (area for 2D)] overlapped with the size distribution of human islets (dashed green line). Insulin release is total insulin release (AUC) of differently sized islets, d = 50 to 350 μm, in response to a single glucose step (15 mM, 30 min) under perifusion conditions that simulate physiological oxygen concentrations ($pO_2 = 45$ mmHg) for free islets (l = 0) as well as thin- ($l = 25 \mu$ m) and micro-encapsulated ($l = 150 \mu m$) islets.





Calculated insulin concentrations as 3D height data. Surfaces are color-coded for oxygen concentration (blue high, red low) for free islets in normoxic (p_{O2} = 140 mmHg) (**A**) and hypoxic conditions (p_{O2} = 25 mmHg) (**B**). The insulin secreting ability of the large islet is more severely affected by hypoxia as clearly indicated by the changes in relative height between A and B (note different scales).



as well as insulin outflux (D) in response to a glucose step-up at various well depths (0.5 to 1.5 mm) for two islets perifused at atmospheric oxygen (E).

References

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